Annual Review of Critical Psychology
2019, Vol. 16, pp. 245-267

Understanding and reinterpreting Piaget

Gisela Ulmann

Bringuier: *Do you feel you've been badly interpreted?*
Piaget: *Yes, in general.*
(Interview with Bringuier 1980, p. 54)

Abstract
"I am not a psychologist. I am an epistemologist", Piaget once declared to Bringuier in an interview, admitting that he was generally misperceived. In order to understand him better, his research work is biographically unfolded. Following this, Piaget's data are reinterpreted within the framework of Holzkamp's ontogenetic categories. It is shown that Piaget's hypotheses regarding the developmental dynamics fail to recognize human subjectivity.

Keywords
genetic epistemology, development of knowledge at children

I. Confusions concerning the "developmental psychologist" Piaget

Jean Piaget began to publish his empirical studies in the 1920s, but was recognised by mainstream psychology only much later. Hans Aebli, a professor at Berlin´s FU in the 1960s, contributed significantly to Piaget becoming known in Germany through the many of his works that he commissioned for translation. In the United States it was above all John H. Flavell (1963), who (albeit

limitedly)[1] brought the Swiss Piaget to greater attention. Piaget was and still is regarded within the discipline of psychology as a developmental psychologist. In the encyclopedic "Psychologie des 20. Jahrhunderts" [Psychology of the 20th Century] published by the Kindler Verlag, Volume VII on developmental psychology from 1978 bears the title "Piaget and The Aftereffects". In the standard German textbook on Developmental Psychology edited by Oerter and Montada (2008) as well as in that of Schneider and Lindenberger (2012) several chapters refer to Piaget and present his theory as that of "intellectual development" or as "development of thought". Yet, is Piaget a developmental psychologist as he is held to be?

II. Piaget´s questions, methods – and his results

Having completed a doctorate in Biology, Jean Piaget began to take an interest in epistemology and in the 1920s decided to become a "psychologist", by which he meant he began to engage in empirical research. From the point of view of Piaget the philosopher, empirical research counted as "psychology". His goal was, however, the development of a genetic epistemology, as Piaget himself maintained in hindsight: "I'm not a psychologist. I'm an epistemologist. (*Still, you practice experimental psychology?*) Because I want facts." (Bringuier, 1980, p. 49). In this respect Piaget´s works would more easily be placed within general psychology rather than developmental psychology. In order to determine the significance for psychology of his research findings it is necessary, in my opinion, to clarify what their subject matter actually is.

Piaget himself describes this in a nutshell: the subject matter of the theory of cognition are "the psychological origins of the notions and operations" of knowledge (GE 1970, p. 1)[2]. Piaget often describes "knowledge" as "intelligence". The subject of a *genetic* theory of knowledge is the genesis of operations and concepts. Piaget also pursued the genesis of "knowledge" historically; since the available evidence appeared to him as insufficient, he

[1] Gopnik et al (2000) write that Piaget's and Vygotsky's work "was almost completely ignored" (p. 19). They, like Astington (1993) point out that it was only in the 1990s that the appeal of Piaget´s methods decisively advanced research in developmental psychology. In the 1960s and 1970s Piaget was indeed much discussed in the USA but his methods were derided as "storytelling", since he neither quantified nor statistically evaluated his data.

[2] Since Piaget´s research is presented here above all as historical research, I always indicate the original year in which a work was first published. The corresponding page numbers of the quotations refer to the English translation. In the bibliography I cite both editions --as far as English translations exist. I indicate Piaget´s work in the article with the first letters of their titles.

turned – like “biologists" who instead turn to the investigation of ontogeny– to children in order to study the genesis of knowledge. “What is wonderful about the child is that you always have an individual starting from scratch, and you can see how all this occurs” (Bringuier 1980, p. 20). "The fundamental hypothesis of genetic epistemology is that there is a parallelism between the progress made in the logical and rational organization of knowledge and the corresponding formative psychological processes" (GE 1970, p. 13).[3]

Piaget begins with the objection that, according to the accepted epistemological conceptions of his time, the categories of knowledge must either precede this, as Kant assumed (apriorism) -- *or* else evolve through experience, as assumed by Hume (empiricism). As a result, he maintains a front within psychology against gestalt psychology as "structure without genesis" and "associationism" in the sense of behaviourism (see PI 1947).[4] Piaget contests both: categories can be neither innate nor acquired through simple experience. He posits instead that experiences are made through active contestation with reality, which become schemata or are reflected in operations; categories are constructed by means of these operations.

The data that Piaget amassed with many colleagues in decades-long research work with children deal with the pre-stages of the operations and categories of knowledge. With this a genesis in the sense of a process is established. Yet the process itself is not visible through the data but can only be deduced by way of interpretation. Before these interpretations are examined, the operations and categories as Piaget conceived of them must be outlined. For this purpose, I adopt a research-biographical approach so that the overall design will be comprehensible and hence clearer.

Piaget commenced his empirical research in the Theodore Simons Laboratory in Paris at the start of the 1920s. Here he did not find the conventional work of constructing and standardizing intelligence tests as particularly interesting, yet he was fascinated by the manner in which children responded to intelligence test questions (A, 1950). He pursued this in greater detail using "clinical methods": he interviewed children and discussed with them. He regarded the ways of thinking that were expressed in the children’s answers to be "precursors" of a developed intelligence, which were also evidence for the genesis of knowledge (LT 1923, JR 1924). In the course of this he also compiled questions for children, which he later posed to children so as to then analyze their answers (CW 1926, CP 1927, MJ 1932).

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[3] I haven´t found any reference in Piaget to Haeckel and his formulation of a "biogenetic basic law", which states that ontogeny is the recapitulation of phylogenesis.

[4] In 1970 Piaget states that the "stimulus-response scheme” is not an association, but an assimilation of the relevant "stimulus" in an existing structure, upon which the reaction follows (PP 1970, p. 707f).

The first decade of his empirical research furnished Piaget with, as it were, two stages of knowledge: a (developed) end-stage of logical thinking that humans acquire from about 11-12 years of age, and a pre-stage that human children put into practice up to this age. In five publications he describes this thinking "in children"[5] most notably as "egocentric" and – in this respect– pre-logical and pre-conceptual. With "egocentric" Piaget means children are not yet able to adequately distinguish themselves as the subjects of knowledge from the objects of knowledge. This "epistemological attitude" does not allow them to differentiate between thinking *about* reality from reality itself, which is why they equate their own thinking with reality. In this respect children initially form theories about the world based on the pattern of their own activities. They see objects thus as living, manufactured and evolving, as having intentions, etc.; in this respect the necessity of evidence is alien to them – and they do not recognize inconsistencies. Piaget later distanced himself from some analyses of his "early work": most notably he had not sufficiently analyzed the category of "causality" (Bringuier 1977 p. 101 f.)

Two very significant events at this time led to a further development in method and hence theory. First, Piaget´s three children were born (1925, 1927 and 1931), whom he and his wife *observed* from their birth with regard to pre-linguistic possibilities for knowledge and then (as soon as possible) *interviewed* them. These observations brought Piaget to the conception of another "pre-stage": knowledge operations are from the very beginning "sensomotoric", that is perceptive, grasping and involve movement, i.e. walking. Piaget published the results in two parallel works: In 1936 "The Origins of Intelligence in Children" appears, in which Piaget, starting from reflex, works out activity schemata in six stages, that is, the precursors of the *structures*[6] of thought. In 1937 "The Construction of Reality in the Child" appears, in which Piaget presents how the *categories* of knowledge (*identity* as a permanent object, *space*, *time* and *causality*) are constructed in the first two years of life by means of the activity *schemata*. A kind of continuation of these observations of his children – in particular the origin of imitation as "pure accommodation" and the symbolic function as "pure assimilation"[7]-- appear in 1945. Only in these three works are

[5] Unfortunately, in the titles of English translations "chez l´enfant" is often stated as "of the child". This very often leads to the criticism that there are children that (for example in the age specified by Piaget) do not think like this, that do not express themselves as such. Piaget pointed out that he discovered these utterances *in children* --and also stated the exact age of the respective children. Every item of data reported by him contains the first letter of the name and the age of the child, i.e. Luc (5;4).

[6] According to Piaget an activity has a *schema* (i.e. shaking/agitating, hitting/beating), an operation has a *structure* (i.e. comparing, forming categories).

[7] On the significance of the concepts of assimilation and accommodation see below.

the observed data also occasionally presented within the living contexts of the children, since after this Piaget altered his methods.

It was another significant event that prompted this change in method: Piaget took into account critiques from the Anglo-american world regarding his "clinical", that is to say, purely verbal method. His new method, which Piaget called the "critical method"[8] consists in him having children solve manipulated problems with concrete material.[9] The "activity" and the "contemplation" enable some children who cannot yet solve purely verbally posed problems to find a solution by means of concrete operations! Yet children who are younger than 7 years old do not use this possibility to concretely manipulate material to find solutions.[10] This prompted Piaget to subdivide the previously homogeneously conceptualised stage of "Child" into one, which he named "concrete operational" and another, which he (in this context) named "pre-operational": There were even children for whom it was not yet possible to perform concrete operations. The first publication in which this new method was applied is "The Child´s Conception of Number" in 1941 from which a series of further publications followed: on the concept of identity (measure, weight, volume 1941b), time, space (1948), causality (1927). His final empirical research appeared posthumously in 1983.

By 1940 Piaget had produced a synopsis and guidelines for further necessary empirical research. This "synopsis" is arguably the first in which Piaget produced an integration of all the "subject matters" he researched referring to data collected to that point, which were only published years later in the respective – and in the meantime expanded– overall context.[11] In it he also speculated on the feelings of children, on which he later, apparently reluctantly, held a lecture (IA, 1954), in order to subsequently finally abandon such "unstructured constructs" such as emotions (Bringuier 1977, 85ff).

What is interesting however about this synopsis from 1940 are two things. First, Piaget also (still) pursued his concern in demonstrating the disappearance of egocentrism, that is, the decentering in the sense of the differentiation between the subject of knowledge and the object of knowledge, but now from infant to

[8] 1947, the forward to the third edition of JR.

[9] In the meantime Piaget worked at the Jean Jacques Rousseau Institute, which was affiliated with a preschool where Piaget could conduct research with many children.

[10] Solving the verbally posed problem: 'Edith is fairer (or has fairer hair) than Suzanne; Edith is darker than Lili. Which is the darkest, Edith, Suzanne, or Lili?' corresponds to a formal operation. Children who cannot solve this problem, but who then can if they are shown equivalent dolls, are able to think concretely-operationally. Children who still cannot solve this problem despite the dolls, think in a pre-operational manner: they cannot yet understand that Edith is fair as well as lighter. (See JR 1924, 87).

[11] A second one --with Inhelder– followed in 1966. As far as I know it was only in this publication that Piaget claimed to have outlined the "psychology of children".

"adolescents" up to adults. A newborn cannot at all distinguish itself as a subject of knowledge from an object of knowledge. Yet even if the "Copernican turn" succeeds, a child can incorporate itself as a body among other bodies, yet it mixes up the perspective, believing that its perspective is in accord with that of all other people. Only later does it become possible for the subject of knowledge to recognize its own position and its own perspective as *its* view of the objects, to coordinate these with the others and finally to categorize itself into the community (of recognizers). These turning points serve Piaget in conceptualizing qualitatively different stages in the development of knowledge.

On the other hand, in this compendium the notion of the genesis of knowledge as Piaget conceives it becomes clear: with each respectively developed activity schemata and operational structures of knowledge the categories of knowledge can be elaborated. For example: once a child can conceive of the unviewed relocation of an object, it looks to where it was last hidden (before its eyes) – objects now become permanent.[12] Quantity, in contrast, is not yet permanent for this child. Instead, it suggests that a ball of clay rolled out into the form of a sausage has now become more (since it's longer) or also less (because it's thinner). Only when it can mentally combine both these changes can it recognize that one and the same activity (rolling it out) makes the quantity thinner as well as longer; provided that thickness and length compensate each other the quantity hence remained constant.

In this first compilation (1940) Piaget still arranged the stages according to age specifications[13]. Later (PI, 1947 and PC, 1966) he arranges according to sensorimotor and operational knowledge. Additionally, he incorporates the development of the symbol function that he had studied by then (PDI, 1945), which in his opinion makes possible the transition from sensorimotor to operational knowledge.

Piaget himself always presented the sequence of steps from the ontogenetic beginning outwards and hence showed how those that follow in each case "transcend" (preserving and evolving from) the preceding stages, from which follows irreversibility. In my opinion, however, this sequence becomes much clearer if we consider the stages retrospectively, that is to say, starting from the most developed form and going backwards to the respective preform. If we

[12] If an object in a closed hand is hidden under A and found there by a child, but then the hand with the object first travels under A, and then under B, the child searches for the object under A, where it previously had found it. According to Piaget the object is thus quasi dependent on the activity of the child, in this sense still not permanent. It is only so (permanent) when it can be seen by the child as being independent from its own activities.

[13] 0-2, 2-7, 7-12 years, adolescence. These age specifications are empirical – not normative!

outline the development in four[14] stages (guided by SPS, 1940), the following results: cognitive capacity, whose genesis interests Piaget[15], is described by him as "*formal-logical intelligence*" (empirically demonstrated by him with children and youngsters from circa 11 years old). Thinking is hypothetico-deductive, hence, independent from concrete reality. Thus, conclusions can be drawn from sentences (propositional logic), whereby sentences can be mere hypotheses. For example, a youngster recognizes: when all a's are elements of the quantity A, and this is an a, then it is an element of the quantity A. *Hypothetical thinking is the particular "ability" that identifies this stage.*

The preform: hypothetical capability is missing, knowledge is dependent on concrete experiences. Thinking is (already) internalized and thus reversible.[16] *Once* concrete operations are possible, then identity (as an identifiable invariant of a particular transformation), space (as the coordination of all coordinates and perspectives), time (as the coordination of the sequence of events *and* the interval between events), and causality (in terms of a specific cause and its effect) can be comprehended. Classes can be formed and arranged according to a uniform feature and placed into a hierarchy; i.e. a child now knows that the quantity of all flowers is greater than that of the bellflowers. Piaget calls this *"concrete operational intelligence"* (empirically shown by him with children between 7 and 11-12 years of age). *The particular "ability" at this stage is reversibility, that is, the mental reversibility of operations.*

The preform: reversible thinking is not yet possible, thinking is "pre-operational" (empirically shown by Piaget with children between 2 and 7 years of age). This follows the one-dimensional activity and the one-dimensional perception of the subject of knowledge, that indeed already distinguishes itself from the object of knowledge, but that cannot yet coordinate its own point of view with other points of view, that is, its perspectives with that of other people. Invariants of transformations are not recognizable (several dimensions such as "length" and "thickness" are indeed discerned, but not yet coordinated), speed

[14] Students have often asked me how many stages Piaget conceptualized. The answer would be: at least three, but according to what he wanted to differentiate there are many more. I refer here to four stages in order to make the logic of development intelligible.

[15] Other than what critics often accused him of, Piaget does not see in formal-logical intelligence "the end" of the development of knowledge possibilities. As for the question whether afterwards dialectical thinking would not yet develop (Kesselring 1988, 163ff), he doubtless would reply that he was too little familiar with dialectic (Bringuier, 1980, p. 100).

[16] Thus a child can for example think about making a sausage out of a ball of dough and *simultaneously* a ball out of the sausage and thus realize that nothing changes through the deformations regarding quantity, weight and volume; these dimensions are hence "identical" regarding the sausage and ball of dough.

still decays in spatial sequence and temporal interval[17], space is not yet consistent (locations and perspectives of respective coordinates cannot yet be coordinated[18]), causality cannot yet be differentiated from intentionality[19]. To be sure, symbols can already be formed, but concepts are formed syncretically (that is, by alternating dimensions[20]) and cannot be thought of in hierarchies. *The particular "ability" here is the "internalization" of the knowledge activity.*

The preform: internalization is not possible, knowledge is only possible through activity and perception, it is contained in it and is not to be uncoupled from this[21] --it is "*sensomotoric*" (empirically shown with children from birth to circa 2 years of age). The subject of knowledge and the object of knowledge cannot be differentiated. Objects can be classified with respect to their application by means of a schemata of action, methods can be applied to purposes. *The earliest possibility for knowledge from birth is the (sucking) reflex, since via the sucking reflex the class of suckable objects can already be distinguished from the class of non-suckable.*

From this deduction it follows that an inversion of the sequence of these (pre-)forms of knowledge possibilities is not at all possible for logical reasons. The formation of each of these stages can nevertheless be delayed or accelerated, as Piaget himself (already in 1936 with regard to his own three children) expressly highlighted.[22]

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[17] Hence a toy train that travels just as long as another but for a shorter distance is not regarded as slower, rather as just as fast --since both set off and arrived at the same time (PC, 1966).

[18] Thus a child holds *its* perspective on a board with three clearly distinguishable piles (of things) for *the* perspective per se, this also applies for an observer who stands across or to the side of the child, and the child cannot specify from what perspective the different photos of this constellation were taken (CS, 1948).

[19] Thus a child believes that a marble that rolls down an inclined board wants to go to the teacher standing under it--even after someone tries to explain to the child what a slope is (LT, 1923).

[20] Hence a button and a coin belong together because they are round; the coin resembles a duck because on the coin a bird is depicted; the bird is equated with water because it can swim in it; the water resembles milk because one can drink both. (This example is attributed to Darwin´s observation; see Vygotski 1987, 176.)

[21] Thus a child regards an object visibly hidden in front of its eyes initially as having disappeared; it does not search for it and cries if in case it wanted the object; somewhat later it searches for it in the place where it last found it. The child does not yet realize that only a specific activity has a specific (desired) outcome. The child in some cases mixes up chronological sequence, it explains for instance that we first blew out and then lit the candles(CR, 1937).

[22] Hence his eldest daughter was the "slowest" --because, born in winter, she spent a lot of time in a baby carriage on the balcony without almost any possibilities for activities (OI, 1936).

Excursus on speed – on phases and stages

Since the still prevailing critique of Piaget directs itself above all on his "too late" starting of the knowledge capabilities of children, in particular with regard to that of a "permanent object", it is useful to look more closely at this. More modern investigations[23] with practices like habituation conclude that since three-and-a-half month old infants already look longer at events that are contrary to expectations than those that conform to expectations, that objects for them also exist when they are fully covered. This corresponds with one of Piaget´s observation of his three-month-old daughter whose facial expression registered disappointment and expectation as her father hid himself. A crucial criteria that a stage has been reached is, however, according to Piaget, if a child *searches* for a disappeared object. The age group that Piaget gives for this to first occur is also observed in modern investigations. Does a child know that an object exists even when it is hidden and it just does not look for it --or does it not know this? The answer lies in my opinion in Piaget´s conception of a "stage" that completes a "phase": this is reached when no more mistakes at all are made with objects of knowledge that have the same structure[24]. Piaget describes the cognitive achievements occurring before this as prestages. That Piaget uses the words phase and stage as synonyms may remain hidden to a perfunctory reader. Piaget remarked on the problem regarding the acceleration of development through experience 1955 (GLT), 1972 (PGP) and 1980 (Bringuier, p. 44). He addressed this systematically in 1959 (TL, 1970). Here he observes 4 types of "apprentissage"[25]: maturation, substantive experience, social transmission and equilibration. Each type contributes to the formation of knowledge, the first three mentioned are however inadequate as an explanation. The objections to material experience lead to the conception of the pre-operational stage. Against social transmission Piaget argues that a child must first assimilate and digest for itself that which someone tries to teach it[26] --what in turn is contingent on the laws of

[23] For a summary see Schneider & Lindenberger 2012, 391.

[24] If a Berlin child would say for instance that there are actually more animals than dogs, but in Berlin there are more dogs than animals, then the stage has not yet been reached.

[25] English translation: learning.

[26] According to Piaget, many of his critics have provided evidence precisely for this. If need be one can teach children to correctly repeat words or sentences, i.e. to imitate, --yet since they cannot apply these to structurally similar situations shows that they do not understand the “influence of the social environment”; they have not made it their own. (PP 1970, p.714ff)

spontaneous development[27]. In this respect, according to Piaget, only the concept of equilibration (see below) is an adequate explanation.

III. Piaget´s conception of the dynamic of development

How does Piaget conceive of the "motor" of development? What advances the process? In general, this is not accessible through simple observation.

By way of illustration: When a plant is photographed daily or filmed continuously, we can indeed see that it grows and produces leaves etc., yet we cannot see why it grows --we do not see that it needs water and light for this, rather we can only come to know this through further investigations and theoretical efforts.

Piaget did not undertake this. He proceeds on the assumption that every subject of knowledge is active, produces experiences through activity and reflects this. His point of reference initially was the differentiation between the subject of knowledge from the object of knowledge (the decentering of egocentrism). Later he switched to the cybernetic paradigm of self-regulation of structures (BK, 1967), as will be further outlined in the following:

In his early writings Piaget proceeds from the view that a "child", because it still cannot distinguish between a subject of knowledge and an object of knowledge, and therefore is not conscious of its own subjectivity, thinks "egocentrically" by necessity. It must then quasi absolutely posit its own location and its perspective which then leads to the egocentric distortion ("assimilation") of knowledge. To the extent that the child de-centers its thinking and can distinguish its thinking about things from things themselves, its knowledge can become "objective"[28]. This occurs on an ever-higher level so that the youth must then understand that even its theories about the world and their refinement differ from the theories of others and the world itself, and in this respect can transform from a reformer into a realizer with respect to Piaget´s tested categories[29].

In later writings, systematically in Piaget´s work "The Development of Intelligence" (1936) the concept of "assimilation" borrowed from biology and – as a countermovement– "accommodation" took on priority as an explanation for the progress of development: already with its innate sucking reflex an infant "assimilates" surrounding factors and "accommodates" the reflex on these.

[27] In my opinion Piaget uses "spontaneous" to refer to the child's own construction.
[28] For Piaget "objective" means that even objective reality is brought to account as an object of knowledge and is not simply thought of as being formed according to the subjective image of it or is open to influence by it.
[29] I will return to this at the end.

Assimilation refers to the utilization of a knowledge schema and a knowledge structure to a set of circumstances, whereby, conditions permitting, it results in a distortive perspective/way of thinking. Accommodation refers to the adaptation of a knowledge schema and a knowledge structure to the set of circumstances. The direction of these processes is determined when they come into balance, what Piaget calls "equilibration". "Balance" means: a specific activity from the standpoint of the subject is from an assimilation; from the standpoint of the object from an accommodation; however, in terms of the result assimilation and accommodation do not differ substantially (Brinquier 1980, 44**)**.[30]

By having replaced the egocentrism-concept in his theorizing with the concept of equilibration (which also has the advantage of being completely non-dependent on age), he also, in my opinion, eliminated the recognizing subject from his theory. It is thus the schema, or, the structure, which assimilate, accommodate and coordinate themselves and equilibrate in this way. Piaget uses the emergence of cybernetics as a model to explain the self-movement of structures (Biology and Knowledge 1967). In Genetic Epistemology (1970) he designates this explanation as being too general and in 1975 puts forward a more comprehensive explanation with the title "The equilibration of cognitive structures", in which he expressly critiques his earliest interpretations of the mechanism of development. As is already indicated in the title, the structures are quasi the subject; and even when Piaget here differentiates the observations of the activity of the subject from the observations on the object, his conception remains one of self-movement. Piaget sees its dynamic as being in the fact that no result of assimilation is fully in accord with a corresponding result of accommodation, insofar as each solved problem still manifests existing contradictions – however, in my opinion, he gives no explanation for why these still existing inconsistencies are (later!) recognized.

Piaget, by substituting the concept of (decentering) egocentrism with the concept of equilibration of structures, no longer poses the question of how the subject of knowledge discovers an inconsistency in its (putative) knowledge, since "all knowledge brings up new problems just as much as it resolves old ones" Piaget (ECS 1975, 25). The children, whose reasons he describes, are nevertheless for a certain time steadfastly convinced of the correctness of their answers.[31] How do they experience "new problems", or, their problem as "new"?

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[30] Thus, an infant that, for example, thus far suckled only a breast, but now sucks on a blanket assimilates this into the category of something suckable – and accommodates its suckling, i.e. its sucking action to the blanket, which is also suitable for this.

[31] Seiler (1968) shows that repeated questioning also does not lead to children altering their conviction. An interpretation for the belief by children that a mass becomes more or less when it is transformed in front of their eyes is found in Ulmann (1997). Children see that it is no longer identical, that is, no longer "equal" --consequently it must have

With regard to experience, Piaget argues against the "empiricists" and against "associationism" that the operations and categories of knowledge presented by him cannot be based on simple experience with objects, rather this is only facilitated by the reflection on the experience. This is made clear with a simple example: When a child is asked whether there are more flowers or more bellflowers, and it initially answers that it did not count these, and it regards this as an empirical problem to be solved, and when the same child somewhat later correctly answers this question, it is not for the reason that it in the meantime the child counted them, but because it can place its operations of category formation *reflectively* in relationship to each other. Yet according to Piaget the movement is "spontaneous" and takes place in self-regulation.

With this conception Piaget clearly differentiated himself from nativism. Viewed from a genetic-psychological perspective he argues rather "constructively"[32] (schemata and structures construct the categories) or "epigenetically", but in my view using a quasi theory of maturation, since he does not pose the question why a subject does not attend to an inconsistency.

This must be problematized. As stated at the outset, Piaget assumed a parallelism between ontogeny and sociogenesis, he also intensively engaged with the history of science, and again and again pointed out that children develop specific outlooks and theories as a rule in the same chronological order as these were developed socio-historically (see for example CQ 1941a Bringuier 1980, 93ff). It in no way escaped him that a child in the 20th century demonstrably progresses much further than an adult from more than 2000 years ago. Yet he did not consider, to my knowledge, that children today can form operations and categories that adult humans from more than 10 000 years ago could not at all![33] This is in a way very peculiar, since Piaget's most superb topic of research were logical-mathematical structures, whose development, on the one hand, count as not yet completed, and on the other hand, in today's known historical documents were only tentatively used circa 4000 years ago. The question to be asked: *Can a child itself "construct" the concept of numbers as Piaget investigated it --or can*

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become more or less. As long as they cannot yet distinguish that "something" is now other, "others" but yet the same, they do not let themselves be dissuaded from this conviction.

[32] In no way "social-constructive"; according to Piaget the categories of objective reality are becoming more and more adequate.

[33] The formation of the abstract concept of numbers that Piaget (CN 1941) empirically presented with children at the age of roughly 7 years, relates to that particular concept that first became possible via "arabic numerals", which in turn emerged social-historically in India between 600-900 CE. Up until then and for some (adult) people even today things are "counted concretely": i.e., by tallies: per a real animal, bread, etc. a notch or indentation. (see for example Ifra 1991).

the child pick up (constructing it) the concept of numbers, because (or if!) it is socially developed?

If we understand ontogenesis as an individual *process within a process* of social-historical development (see Holzkamp, 1983, Chapter 8), then only the latter can be a correct answer. The possibility of thinking is not the only thing inherent to children, which Piaget – as "is universally accepted" – is in agreement with (PDI 1945, 196), but also the possibilities for the *acquisition* of respective social-historically developed knowledge – yet this also precisely requires its social-historical development. If one does not assume that this social knowledge can be instilled into children, as Piaget rightly rejects, and if one also does not assume that a child isolatedly deals only with current facts, objects, circumstances ("reality" according to Piaget) and actively experientially grapples with them, but rather when we take into account that a child is also confronted with *theories about these* and reflects these, we come to a better understanding of the process of acquisition that at times appears as "spontaneous development"[34]. It is from here that we can also evaluate the psychological relevance of the data presented by Piaget.

IV. Re-interpretation 1.: Development: the subject "climbing" over steps

If we only consider the data as Piaget comprehended it in the 4 phases described above, his research objective becomes clear; this should also be clarified by means of a developmental-logical reconstruction. He proceeded with a stage defined by operations and categories of knowledge and pointed out logically-subordinated pre-stages. He explained this not only theoretically and logically but also provided empirically supporting data for it. Yet he by no means describes the "intellectual development" (let alone the cognitive development) of children (which is usually asserted in psychological receptions), rather he only illustrates the *sequence* in which children acquire, in the actual sense of the word, these social-historically developed operations and categories; how they are able "to make it (with understanding) their own, *if* they have the opportunity (and the ability) for it.

They do not even need to do this. This is clear with the concept of numbers and mathematics: If the concept of numbers and addition, as well as

[34] This is how the following episode was presented in our project "Subject development in early childhood": two children quarrel over which of them is bigger. Na points out that she is 5 years old and that No is only 4. No points out with gestures to her taller height --and a little later differentiates ''bigger" regarding an older age and taller. Words that signify constructs and theories which these children acquired made possible the differentiation they needed for the resolution to the conflict and disagreement.

multiplication, were not socially developed no child would be able to "develop" them out of itself. Yet, although the concept of numbers and mathematical operations are socially developed, a child can nevertheless fail in the attempt to acquire it or even choose to refuse to do it. That the acquisition of the concept of numbers *must precede* the acquisition of addition, and to that the acquisition of multiplication, is "logical", because addition requires the concept of numbers; multiplication is an abbreviated or special form of addition (i.e. 4+4+4+4 = 4x4), which in turn is the potentiation of a special form of multiplication, etc. The inversion is not possible – yet it is still possible not to acquire all this.

Piaget's indisputable service to psychology, in my view, lies in the fact that he, as it were, was a master in "skeletonizing" the operations and categories of knowledge that he investigated and then making relevant observations regarding their sequential (pre-)forms. He was also able to invent experimental designs, which made the structure and the coordination of the categories discernable. Through his "epistemological lenses" he could see this razor sharp – but only this. He himself incidentally noted that formal logic is "not an adequate description for the whole of living thought; formal operations constitute solely the structure" (PI, 1976, 150).

V. Attempts at application – and misunderstandings

An "application" that Piaget himself suggested in 1947 (PI, 154-55) is the individual diagnosis. At the time he pointed out that his colleague Bärbel Inhelder, by screening using the concept of invariance could differentiate the slightly "mentally retarded" from the "feeble-minded" and these from "imbeciles". Although later he no longer returned to this. His (problem) questions were occasionally applied (unstandardized) in the assessment.

Nevertheless, it should be assumed to be a mistake that the acquisition of knowledge is fulfilled by all children exactly like this, as Piaget described.[35] As strongly emphasized, Piaget looked for and found "preforms" *among* children; one single proof for a developmental-logical preform would have sufficed. That Piaget observed, interviewed and experimented with more than one child was quite necessary in order to exclude that what was being dealt with was an individual peculiarity with each observed preform. Guided by the concept of "normal" as described by Alfred Binet, Piaget proceeded from the assumption that something is "normal" for a particular age group when it can be observed in

[35] In my opinion Piaget took responsibility for responding to this "mistake" since he described *every* prestage as necessary, "It's the same because each stage is necessary to the following one. It's called a 'sequential order'" (see Bringuier 1980**,** 25).

circa 75 percent of the children (JR, 1924); percentages require larger quantities. Piaget indeed emphasizes "sensorimotor intelligence" as the first preform. He sees the structures of operations and concepts already primed in the modes of activity and perception, and therefore as *necessary* prestages of logical thinking (Bringuier, 1980, 25). Yet it would be an error to assume that a child *must* pass through this prestage to acquire logical thinking and the categories of identity, space, time and causality. Even if, in general, "grasping" precedes grasping as "understanding", this in no way means that the ability to (physically) grasp something is a necessary prerequisite to be able to understand. The same is true with respect to seeing and hearing – and recognizing. Physically handicapped children find other verifiable ways.[36]

Misunderstandings regarding the application of Piagetian findings exist also in my opinion with regard to mathematical didactics; when examination questions are used as teaching-learning tasks. Piaget worked out that being able to count does not mean being able to recite numerals in the correct sequence, but rather that the concept of numbers is instead formed by the assignment of seriation (ordinal number) and classification (cardinal number), and that it is preceded by a pre-operational way of thinking in which quantities are considered according to their spatial extent: i.e., children judge that two rows with 8 small plates each, which are however of different "lengths", contain different quantities. From this it is concluded that a child must first acquire the concepts of "same, more, less" in the pre-numerical sphere, before it can acquire addition and subtraction. In the respective textbooks,[37] instead of numerical quantities being explained to them, children are hence initially confronted[38] with just such tasks, with which according to Piaget's research in some cases they initially have difficulties, (see in addition to this Ulmann 1992 and 1997).

A *useful* "application" for the everyday practical interaction of adults with children consists, in my opinion, in a better understanding of precisely this. When one knows that a very young child perhaps cannot yet take the point of view of someone else, then one will not interpret it as being "mischievous" when it gets into the hair of another child. Hair is so very nicely soft and if one pulls on it there are sounds to be explored. The "social behaviour" of a very young child

[36] Vygotski (1924/1975) points out that there are different ways for the schooling of socially developed abilities, all of which are cultural, whereby the usual way is seen as being "natural" and all the others, as it were, "artificial". Learning cursive handwriting is considered to be normal, acquiring the ability to write braille is considered an indirect route. Piaget sees, as it were, only the path that children set out on without any hindrance. How deaf children develop knowledge was studied by Furth (1966).

[37] See Panknin, M. et al. (no date given) Arbeits-Diagnose-Förder-Blätter. Mathematik. Senator für Schulwesen Berlin.

[38] Seiler (1968) has shown that no progress occurs for children even when they are repeatedly subjected to such exercises.

can also consist in it bringing another crying child its toy; or in it likewise crying; or that it hits the crying child (perhaps because crying and hitting "belong together" and the blow is yet to come – or because that is how the noise stops). When one knows that specific "mistakes" in the thinking of children are present at specific ages then one can understand children better. When for instance a thirsty child stubbornly fights to get only the slender glass among many glasses with the same amount of juice (because in this one the liquid stands at the highest) even when one assures it that all glasses contain the same amount, one will not regard the child as stubborn but rather understand that it only notices the height of the liquid and from this it infers the quantity.

VI. Re-interpretation 2.: The data

There is a significant difference between experimental settings and life praxis. In the 1930s Vygotski (1969) criticized Piaget on account of the lack of life praxis in his research. Piaget only got around to reading this criticism almost three decades later and responded in 1962. He regretted that Vygotski, who died long ago, could not know the results he derived with his "critical method". Distorting clay balls or threading beads (activities which Piaget listed in his response) are indeed "practical" activities (in contrast to verbal) yet they are in no way what Vygotski meant by life praxis. What was intended was made quite clear, in my opinion, in a report on expeditions undertaken by his colleague Alexander Luria in the 1930s.[39] People, whose (rural agricultural subsistence) life praxis did not make a formal logic necessary, also did not "develop" this at a much older age according to Lurija's observations; they formed concepts "situationally". People who received at least a minimum of formal education, or who worked for some time on collective farms, could construct formal categories and draw logical conclusions.

Analytical categories for the itemization of Ontogenesis as a life praxis were conceptualized by Holzkamp (1983). It is worth testing whether, with this approach, the psychological content of some of Piaget's empirical data can be adequately recognized – and whether this is even data, which Holzkamp applies to the actual empirical research by means of the analytical categories conceptualized by him.

Holzkamp at first reconstructs analytical categories phylogenetically and then socio-historically. He does not conceptualize formal-logical intelligence, as does Piaget, as a central category, but rather "action potence" as a *psychological*

[39] The expeditions took place in 1931 and 1932 (Lurija 1976); a lengthy publication appeared only in 1974. A German translation appeared in 1986.

aspect of a socially mediated mode of life. This – derived phylogenetically and then socio-historically– serves him as a point of departure to achieve the categories for the itemization of ontogenesis up to the attainment of the ability to act. The ability to act requires the (thinking) *transcending* of the *immediate* circumstances of life and life *praxis* towards the recognition and consideration of social structures that cannot be vividly experienced.

The possibility to thinkingly transcend immediacy must be preceded by an ontogenetic process, by which, in the (understood) transcending of immediacy, the socially mediated mode of life/life praxis first becomes perceptible. Holzkamp expressly supplies no data on ages for this "developmental course"[40] of *immediacy-transcendence*, which then changes into a "process-type". This is due mainly to the fact that a particular "essence" is "recognizable" in its social-historical "emergence" and, to that extent, also in its alterability (including one's own way of thinking).

This also implicitly involves recognizing what is genuinely unalterable such as physical, chemical and biological causality, which unlike social laws cannot be abrogated through a legal ruling but can only be taken into account and be used.

An ontogenetic process must precede this process-type, in whose way of thinking the social mediatedness of its own existence is still lacking: that is, a cooperative form of life in which it is already "realized" that objects are in general produced by people, generally for a specific general-human purpose, and for this reason generally have a specific usefulness ("object-intentionality"); this can be realized by understanding that people generally have intentions ("social intentionality"). Holzkamp labels this course of development as involving the *generalization of the meaning of objects* "Gegenstandsbedeutungsverallgemeinerung", which then also transitions into a process-type, and for which Holzkamp similarly emphatically provides no specifications regarding ages.

Prior to this there is only a relative *human-nonspecific precursor* to conceptualize, in which the objects of utility without their social character can only become meaningful via their material characteristic and form, corresponding to possibilities of action or usage.

Holzkamp reconstructs the development of the *human Specific* – Piaget regards this to some extent as pre-existing, since he investigates a specifically

[40] "Developmental course" refers to the process in which a (cognitive) possibility develops. When this becomes dominant Holzkamp speaks about a "process type". "Process-type" approximates in my opinion Piaget's "stage" or "level". "Process" emphasizes however that every new structurally similar problem must be solved --if it even can be solved. Whoever has understood that all are more than a few can apply this on all super- and sub-categories.

human knowledge-possibility and in this respect does not have to pose this question. Yet, as will be shown, this distorts knowledge for him:

Like Piaget, Holzkamp proceeds with an active child, yet with one that wants to improve its life in a contradictory society – and it can do this mainly because it acquires control over its life circumstances by, at first, resolving ontogenetically induced inconsistencies. To this end it must first by necessity "recognize" a lot, which – as very briefly shown– has nothing to do with Piaget's concept of knowledge, and in this respect almost works in the opposite direction, because it is not the *invariants* of the activity which are *primary*, but rather the *variants*. This will be investigated much closer below.

Piaget compiled his data mostly in quasi-experimental settings, which result from his epistemological considerations – and the observations of his biological children also took place through epistemological lenses (he could not research for long with a baby that was "too hungry"). If one reads in particular the observations dealing with the life praxis of children using "psychological lenses", one encounters something quite interesting, for instance, with the data pertaining to the acquisition of identity and causality. This can be shown with some examples:

1. Piaget spotlights a significant recognition problem with children between the ages of 7-9 with their orientation on the "success" of their activities and their modes of one-dimensional perception as well as their one-dimensional "doing" thinking. As long as a child – according to Piaget– is interested in the success of its activity, it is not interested in how this comes about,[41] and as long as it thinks "in doing" (and "in seeing"), it accentuates the changes within transformations and disregards the invariants, that is, that which does not change in a transformation. Furthermore, it *egocentrically* "over determines" its own activity as being experienced as *intended* and also ascribes intentions to things. Thus children believe, for instance, that marbles *want* to roll to a specific location and that mountains, rivers and wood were *made* by humans for human purposes (see in particular CW 1926 and PDI 1945). Piaget describes these childish theories as animism and artificialism.

Yet this "recognition problem" signifies – if read differently by interpreting the shortcoming as an ability– that children are primarily interested in how one can improve life's circumstances, and that in this respect they do indeed acquire knowledge! Thus for a child it is extremely important to understand that people generally – like itself– have intentions, not only because it can then distinguish between the meaningful, and as the case may be, essential activities of others and their arbitrariness, but also, because this makes possible for the child a more

[41] Piaget was able to show empirically (SU, 1974) that a child/youngster initially does not understand its successful practical resolution of a problem.

conscious interweaving of intentions, that is to say, communication with others. And for a child that wants to acquire control over its life circumstances it is quite important not only to realize what objects are used for and how one can use them as an instrument (the sensorimotor phase according to Piaget), but rather for what purpose they were made. Because when, for example, a child ruins things through unsuitable use, they will, if need be, all too quickly be taken away from it.

Piaget's data show: the first attempts to influence the actions of other people are already observable in the second year of life (see 1936 and 1937).[42] That children in this age group also attempt when prompted to make objects do something, which is understood by Piaget as "universal intentionality". What constitutes an "over-determination" with respect to objects, as Piaget describes it, is with regard to people quite correctly recognized! Piaget concludes from his data that the idea of universal intentionality differentiates itself from the understanding of rules on the one hand in "precausality", which leads to the concept of causality, and on the other in psychological motivation, which leads to logical reasoning (LT 1923, MJ 1932). Which types of control (Verfügungsmöglichkeiten) accrue to a child if it understands psychological intentionality – "social-intentionality" for Holzkamp–, Piaget did not consider, rather he only took account of (egocentric) mistakes. He observes this also in the data published above all in 1926, 1927 (and 1945), which concern childish theories about nature. In 1940 he summarized this as follows: "In other words, there is no chance in nature, and everything is 'made for' man and children according to an established and wise plan with the human being at its center. Thus it is the *raison d'etre* of things" (SPS 25). Indeed this is valid for most things that surrounds a child! A child that recognizes this – "objective intentionality" according to Holzkamp – expands its control capabilities. This data from five early publications and three subsequent works illustrate the developmental course of the *generalization of the meaning of objects* (Gegenstandsbedeutungsverallgemeinerung) conceptualized by Holzkamp. The data also empirically demonstrates Holzkamp's notion of a "logically" reconstructed sequence, whose assertion is only possible actual-empirically: at first a child only recognizes the usefulness of a thing; as soon as it begins to recognize human intentionality, it also recognizes that one can make, produce and change something; both lead to the recognition of the usefulness of

[42] About one to two decades after the publication of "Grundlegung der Psychologie" [Foundation of Psychology] by Holzkamp, to which I refer here, but without reference to it, the ontogenesis of the understanding of intentions became the object of research in mainstream psychology (see Astington, Orig. 1993). Tomasello in particular examined this closer and termed it the "nine month revolution" (2002, 61f).

(manufactured) objects. Being able to differentiate manufactured objects from "nature" is the next step.

2. According to Piaget the "sensorimotor recognition pre-form" is outgrown when action can be internalized; yet even for actions only to be thought about requires that their consequences be anticipated. This begins with delayed imitation; later a child can form symbols (symbolic play), its schemata become operations that are reversible but still require concrete materials, until they also become detached from them, become "formal" and make hypothetical-deductive thinking possible. Thus can the categories of identity and causality, etc. also be constructed. A constantly changing world in which, furthermore, nothing is foreseeable can also appear unsettling to children. To that extent their control also obviously expands when they recognize what remains constant, and which specific cause has which specific outcome.[43] This also means, according to Piaget, that a child can increasingly overcome being imprisoned in immediacy (Unmittelbarkeitsverhaftetheit) in all vividness to the benefit of sheer (logical) thinking, that is, logical inference from premises. Viewed differently one could also express it as such: The child/youngster can increasingly "conceptually grasp" his/her practical experiences, i.e. in general terms. This corresponds to an aspect of Holzkamp's conception of the developmental course of transcending immediacy (Unmittelbarkeitsüberschreitung), which holds as essential that socially mediated existence is also recognized. Since Piaget, following Rousseau, understands society as a social *contract*, that is, as an accord between all individual members of a community, this notion does not occur to him.

VII. Conclusion

To summarize: According to Piaget's data a child is primarily above all a "practitioner"; if praxis is understood in Marx's sense as *change-causing* activity: "The philosophers have only *interpreted* the world, the point is to *change* it" (Marx, 1845). As a "practitioner" it accentuates above all the variants, that is, the *changes* via activity – that in certain circumstances it also wants to cause. Presumably because Piaget methodically precluded the life praxis of children, since he only observed and questioned children under quasi-experimental conditions, he could not see, whether and how the *abstraction of life-practical experiences*, as well as the social-historical theories that mediated them, also facilitates their formal thinking and recognition of invariants.

[43] Piaget's data on concrete-operational and formal-operational modes of thinking have never been refuted to my knowledge – even though many children today reach the stages earlier.

Piaget states: "Equilibrium is attained when the adolescent understands that the proper function of reflexion is not to contradict but to predict and interpret experience." (SPS, p. 64) "True adaptation to society comes automatically when the adolescent reformer attempts to put his ideas to work. Just as experience reconciles formal thought with the reality of things, so does effective and enduring work,[44] undertaken in concrete and well-defined situations, cure all dreams." (ibid, p. 68)

Piaget conceptualizes people – according to his own image and his research question– as experimenters, without considering that they must also eat, that food must be produced, and that, indeed, people are *also* producers, they can evolve in how they relate to their world – and must do this.

Yet to understand their socially-mediated existence, another kind of transcending of immediacy (Unmittelbarkeitsüberschreitung) is required than the formal-logical one investigated by Piaget: they have to understand the difference between visible social relations (that correspond to Rousseau's social contract conception of society, which Piaget adheres to) and non-visible social structures. The recognition of social structures did not interest Piaget the epistemologist.

Of course, only when they are "grown-up" do humans, who are no longer children, have the ability to actually influence change in social relations, not only as experimenters.

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[44] In 1940 Piaget also mentions "professional work"; in 1955 he defines "work" as organizing a systematic plan for research instead of just engaging in trial and error, and continually experimentally varying two or more variables simultaneously.

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Gisela Ulmann, Dr. phil. (with habilitation), lecturer at the Free University Berlin. Research focus: developmental psychology and educational psychology.